A Model Based Predictive Control Algorithm for Building Temperature Control

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Abstract—**The paper presents a model-based predictive control algorithm that uses a limited number of control sequences for on-line simulation of future behaviour of the process. Each control sequence used in simulation generates a predicted sequence of the output signal. The predicted output sequences are analysed and evaluated and then, using a set of rules, the 'optimal' control signal is computed. To simulate the future behaviour of the process it is used a process model and also the previous sequences of the input and output signals from the process. The algorithm permits directly use of the nonlinear model of the process. The algorithm is used for simulation of the temperature control in a house and compared with the usual algorithms of type PI.**

Index Terms—**temperature control, model based predictive control algorithm.**

I. INTRODUCTION

Reducing and optimization of the energy consumption in the residential sector is an important issue in the context of the global warming effect. An essential step in this direction is the implementation of a measuring system and monitoring of the electrical energy consumption. This thing can lead to a better usage of the different electrical consumers. In the same time are necessary strategies that take into account the changing (optimization from the point of view of electrical consume) of the user behaviour. In this context, it's necessary the realization of a simulator that will permit the study of different strategies for reducing the electrical energy consume[1]. As it's known, the main part of the energy consume of a house is represented by heating. From this reason, a first step is realization of the thermal model of a house. In literature are presented many examples of modelling and simulation of energy consumption in a household [2], [3] [4]. Thermal model can be used also for the study of some systems HVAC (Heating, Ventilation and Air Conditioning) with electrical heating. Also, the simulator is needed to give solutions in the implementation phase of the project (living houses) and can be used also by the final users. A simulator can be use throughout the whole development phase. It can be used as well for the studies of control strategies (classical, fuzzy, genetic, neural network, model based predictive control etc.) as well as for finding the solutions for reducing the electrical energy consumption and for maintaining acceptable indoor air conditions related to thermal comfort. Also, the reduction of the electrical consumption as well as the aspects that belong to the thermal comfort may be included in the control laws, the main objective being maintaining thermal comfort within an acceptable range.

Testing of the system by usage of a simulator can introduce a series of problems:

- what are the measurable variables and which are the controlled variables (temperature, humidity, air quality, natural light / artificial, etc.);
- the controller is considered as being an independent component or such a combination that includes the sensor, actuator or other elements;
- where and how has the control loop to be divided;
- where is the controller's sensor placed;

A simulator provides an enormous flexibility for the user in order to test. There are various applications possible by simulation: one can develop new control strategies or algorithms for different buildings, one can optimize the parameters of the controller for a given simulated building, etc. There are some major criteria for used simulation tools: validity of the simulation models, easy parameterization, easy comprehension, modularity, possibility of modification, and possible interconnection with other models, hybrid simulation (mixing of discrete and continuous models).

Also, the simulator presents some risks: test of controller under unrealistic conditions, error introduced by breaking up of the real control loop (input/output interface), insufficient level of modelling of system components in the control loop (emitter, building model, sensor etc.), risk of false interpretation due to extremely large number of available data.

The house model can have different levels of complexity: from simple "well mixed" models with one air node representing the whole air volume to complex computational fluid dynamic (CFD) models that take into account the conservation equations of mass and energy.

Models that can have several types: well-mixed models (model of convection), CFD models (model of convection), zonal models (model of convection), lumped parameter models (model of room including envelope), model using identification (convection model or room model).

In the control field there are typical simplified models using supplementary identification or correlation. The necessary phenomena can be modelled by identification with measurements. The system can then be represented as a state space model with the parameters obtained by off- line or on-line identification.

Other type of model is lumped parameter model which have the advantage of a low number of parameters. A set of a few parameters describes the system. A lumped parameter model can integrate all layers of one envelope element (wall, floor, roof, etc.), all elements of the envelope of a room or the whole room model (convection, conduction and radiation in a room). The latter is currently used to simulate rooms in controller studies. In the same way, one or more envelope elements can be modelled as a lumped parameter model. This modelling permits fast simulations since the system is reduced to a fist order system. The model can be described as by thermal-electrical analogy.

II. The Thermal Model of a House

The biggest part of energy consumed in a house is used for heating. From this reason is important realization of a thermal model as detailed and precise can be, thus the simulator can offer solutions for reduction of energy consumption. In this paper is used the model presented in [2]. Similar models are used in [6], [7]. Thus a model can be realized in Matlab/Simulink. In Fig. 1 is presented the Simulink model of a household [2]:

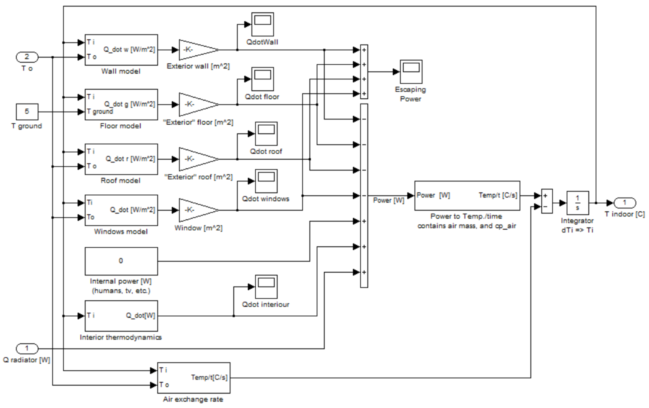

Fig.1 The model of a building [2]

In Fig. 2 is presented the model of an external wall [2].

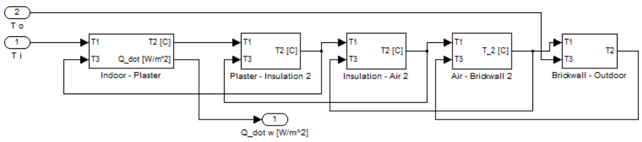

Fig.2 Simulink model of a wall [2]

Since the model will be used by the MBPC algorithm for prediction calculus, it was preferred direct discretization of this model. For example, the Simulink model of the first layer of the wall is presented in Fig. 3.

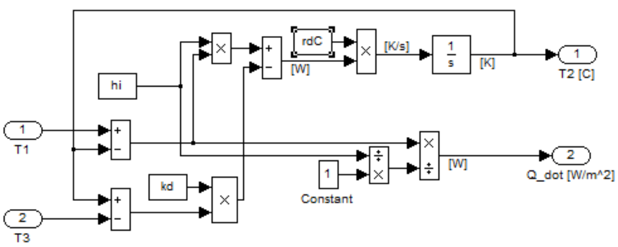

Fig.3 Simulink model of the 1st layer of the wall [2]

This model can be changed into a discrete model thus:
Notations: $rdC=1/(cp1*ro*d_1)$; $k_d=k_1/d_1$;
Dynamical equation:

$$\frac{dT_2}{dt}=\frac{1}{\mathrm{cp}_1*\mathrm{ro}*\mathrm{d}_1}\left(hi*(T_1-T_2)-\frac{k_1}{d_1}*(T_2-T_3)\right) \quad (1)$$

where: ro = density; cp_1=specific heat capacity; d_1=thickness; hi=heat transfer capacity; k_1= thermal conductivity; If T is the sample period then:

$$T_2(t+T)=T_2(t)+\frac{T}{\mathrm{cp}_1*\mathrm{ro}*\mathrm{d}_1}\left(hi*(T_1-T_2)-\frac{k_1}{d_1}*(T_2-T_3)\right)=$$
$$=T_2(t)+ct_1*(T_1-T_2)-ct_2*(T_2-T_3) \quad (2)$$

In similar way, are obtained the discrete models of other blocks and in the final the thermal model of the household.

III. The Control Algorithm

Use of the control system is that if you provide better information to people then they will make better decisions. The goal is to enable and encourage users to make better judgements with regard to energy consumption and emissions. The output data from the household sensors needs to be standardised to allow the control system to handle a variety of household types. The control system needs to be generic in its design. A simulation model using Matlab/Simulink would enable various models to be evaluated and tested. Research needs to be carried out to identify other examples of energy management control systems.

Today, many industrial systems are still controlled by simple PID (proportional-integrative-derivative) algorithms, despite the better performances usually provided by systems developed following the modern control theory. PID controllers can be used to control a wide range of different processes, need only rough process models to be easily tuned and give pretty good set-point tracking performances. On the other hand it is clear that PID performances, although satisfactory, could be improved when dealing with highly nonlinear processes, or processes featuring unmodelled dynamics and external disturbances.

A Model Based Predictive Control (MBPC) algorithm is described by using a model to compute the predicted process outputs. The parameters of the model are obtained through an identification algorithm. Also, a cost function related to the closed loop performance of the system is defined, and the control signal is obtained by means of minimization the cost function. Finally, the first of these signals is applied to the process [7].

The extension of linear MBPC to nonlinear processes is straightforward at least conceptually. But there exists some difficulties [8]: the availability of nonlinear models due to the lack of identification techniques for nonlinear processes, the computational complexities, the lack of stability and robustness results.

The purpose of the controller is typically to force the output to follow the reference signal. If reference is a constant, the problem is commonly referred to as set-point regulation. When the reference is time varying (but is known in advance), defining a control law to force the output to follow the reference signal is called the positioning control.

The basic idea of the algorithm is the on-line simulation of the future behaviour of the control system, by using a few candidate control sequences [9]. Then, using rule based control, these simulations are used to obtain the 'optimal' control signal.

In [10] it was proposed an algorithm (MBPC-A1) designed for setpoint regulation problem (but setpoint can be arbitrary changed). The main idea of the algorithm is to compute for every sample period:

- the predictions of output over a finite horizon (N);
- the cost of the objective function,

for all (hypothetic situation) control sequences:

$$u(.) = \{u(t), u(t+1), .., u(t+N)\}$$

and then to choose the first element of the optimal control sequence.

For a first look, the advantages of the proposed algorithm include the following:

-the minimum of objective function is global;
-this algorithm can be easy applied to nonlinear processes;
-the constraints can easily be implemented.

The drawback of this scheme is an unrealistic computational time. Therefore, the number of sequences must be reduced. Of course, this will lead to some difficulties in finding the global minimum of objective function. Choosing the sequences has to be made with attention, thus through simulation to be obtained information more helpful for computing the control signal.

For a first stage, we used the next four control sequences:

$$\begin{aligned} u_1(t) &= \{u_{\min}, u_{\min}, .., u_{\min}\} \\ u_2(t) &= \{u_{\max}, u_{\min}, .., u_{\min}\} \\ u_3(t) &= \{u_{\min}, u_{\max}, .., u_{\max}\} \\ u_4(t) &= \{u_{\max}, u_{\max}, .., u_{\max}\} \end{aligned} \tag{3}$$

where $u_{\min}$ and $u_{\max}$ are the accepted limits of the control signal, limits imposed by the practical constraints. These values can depend on context and can be functions of time.

Using these sequences results four output sequences $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$. The control signal is computed using a set of rules based on the extreme values $y_{\max0}$, $y_{\max1}$, $y_{\min0}$, $y_{\min1}$ (fig. 4- d is dead time, t_1=N, y_r is setpoint) of the output predictions.

In the followings, considering processes with positive sign, it can be put in evidence four usual cases:

Case 1: If $y_{\max0}<y_r$ (corresponding to $u_1(t)$ sequence) and $y_{\max1}>y_r$ (corresponding to $u_2(t)$ sequence) Then (using a linear interpolation):

$$u(t) = \frac{u_{\max} - u_{\min}}{y_{\max1} - y_{\max0}} y_r + \frac{u_{\min} y_{\max1} - u_{\max} y_{\max0}}{y_{\max1} - y_{\max0}} \tag{4}$$

Case 2: If $y_{\min0}<y_r$ (corresponding to $u_3(t)$ sequence) and $y_{\min1}>y_r$ (corresponding to $u_4(t)$ sequence) Then (using a linear interpolation):

$$u(t) = \frac{u_{\max} - u_{\min}}{y_{\min1} - y_{\min0}} y_r + \frac{u_{\min} y_{\min1} - u_{\max} y_{\min0}}{y_{\min1} - y_{\min0}} \tag{5}$$

Case 3: If $y_{\max0}>y_r$ Then $u(t)=u_{\min}$ (6)

Case 4: If $y_{\max1}<y_r$ Then $u(t)=u_{\max}$ (7)

In fig. 4, every output prediction curve is marked with a number which correspond to the number of control sequence from relations (3). Similar to case 3 and case 4, there are two similarly cases if dy/dt<0 for t<t0.

If the algorithm uses only these 6 rules, the variance of u(t) will be large [10]. So, in the second stage, to limit this variance, depended by behaviour of the control system, are used next methods:

-an algorithm that modifies the limits of control signal:

$$u_{\min} \le u_{\text{minst}}(t) \le u(t) \le u_{\text{maxst}}(t) \le u_{\max}\ ; \qquad \Delta u_{\min} \le \Delta u \le \Delta u_{\max} \tag{8}$$

For example:

$$u_{\text{minst}}(t) = f_1(u_{\text{minst}}(t-1), u_{\text{maxst}}(t-1), y(t), y_r(t)) \tag{9}$$

$$u_{\text{maxst}}(t) = f_2(u_{\text{minst}}(t-1), u_{\text{maxst}}(t-1), y(t), y_r(t)) \tag{10}$$

where f_1, f_2 are functions which decrease or increase (depended by behaviour of the control system) the difference between $u_{\text{maxst}}(t)$ and uminst(t). In relations (3).. (7), the values of $u_{\max}$, $u_{\min}$ are replaced with $u_{\text{minst}}(t)$, $u_{\text{maxst}}(t)$. In the following, if is necessary, the next relations are used:

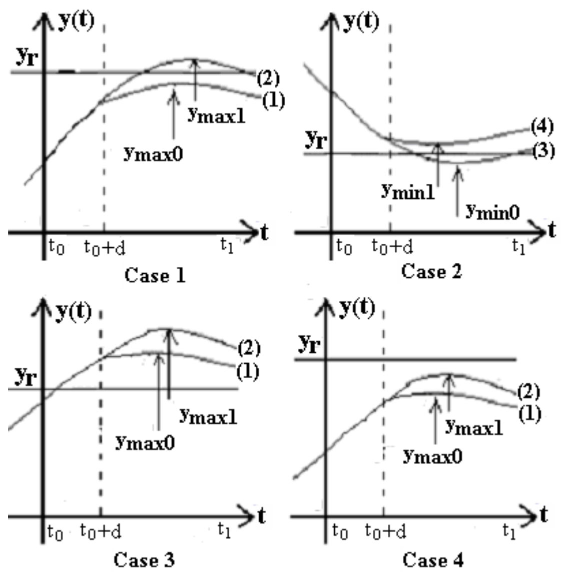

Fig. 4. Examples of output predictions

$$u_{\text{minst}}(t) = u_{\text{minst}}(t-1) + k_{st}(u_{st} - u_{\min st}(t-1)) \tag{11}$$

$$u_{\text{maxst}}(t) = u_{\text{maxst}}(t-1) - k_{st}(u_{\max st}(t-1) - u_{st}) \tag{12}$$

where k_{st} is a weight parameter and u_{st} is the estimated value of control signal in steady state. But in some circumstancing (perturbations, inaccurate model) the limits of control signal must increase. Also, it is necessary to limit the minimum value of $u_{\text{maxst}}(t)-u_{\text{minst}}(t)>d_{ust}>0$, where d_{ust} is a parameter of the control algorithm.

-using the "variable setpoint"[9]:

$$yr1(t)=yr(t)+kref[y(t)-yr(t)] \tag{13}$$

where k_{ref} is a weight factor;

- using a filter to compute control signal (especially in steady state regime).

In what it follows we will consider some examples that will show the working method of the algorithm as well as the choosing of some parameters. For this it is chosen a linear process (P1) of type:

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - a_3 y(t-3) + \\ + b_1 u(t-1-d) + b_2 u(t-2-d) + b_3 u(t-3-d) \quad (14)$$

where y[.] is the process output, u[.] is the controller output, 0≤ u[.] ≤ 250, A[.]=[1 -2.43492 1.97629 -0.53468], B[.]=[0.000948003 0.004438182 0.001296496], static gain is k_0=1; d=1 is dead time. In figures 5..10, are presented non-dimensional the signals of input-output (input, output, setpoint) as well as diverse parameters versus number of sample period.

Example 1

In this example, $y_r[0]$=0 and $y_r[t]$=150 for t>0, u[0]=0 for t≤0 and u[t]=150 for t>0. For PID tuning, it is used Ziegler-Nichols criterion (fig. 5). This example shows the advantages of MBPC-A1 algorithm, comparatively with PID algorithm: a shorter time response, no override. A possible disadvantage is the larger variation of control signal.

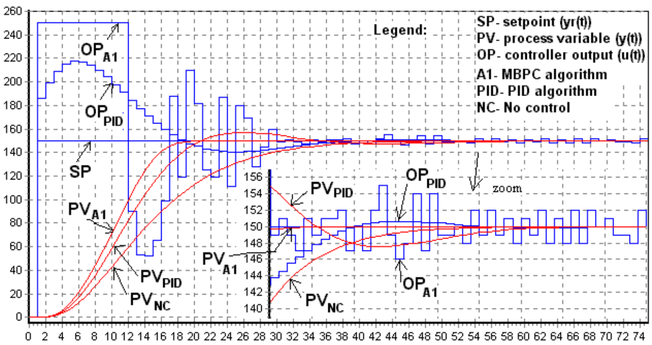

Fig. 5. Example 1

Remark: In the next figures, only the setpoint (SP-setpoint yr(t)) and process's output (PV- process variable y(t)) are represented at true scale. Controller's output (OP- u(t)) is represented as u(t)/3.

Example 2

In this example (fig. 6) the setpoint has a variable shape, the model is accurate (non-adaptive case), k_{ref}=0.2, d_{ust}=5, it is not used information about setpoint changes. This example shows the effect of parameter k_{st}.

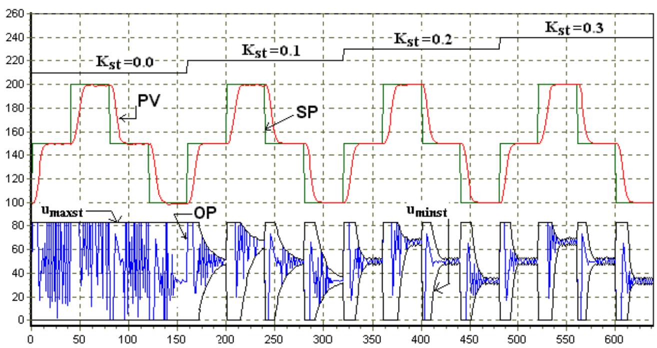

Fig. 6. Example 2

Example 3

Conditions: analogous with example 2 but k_{st}=0.0. This example shows the effect of parameter k_{ref}. In this case (fig. 7), the time response is minim, but the variance of u(t) is larger.

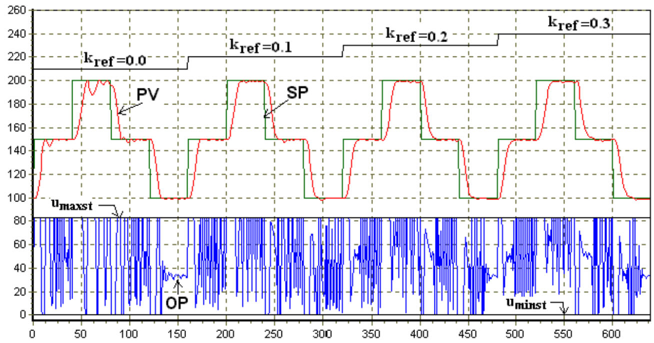

Fig. 7: Example 3

Example 4

Conditions: analogous with example 3 but k_{st}=0.1. This example shows the effect of the value of k_{ref} if $k_{st} \neq 0.0$. In this case, in steady state, the difference $u_{maxst}(t)$-$u_{mnist}(t)$ will decreases (Fig. 8).

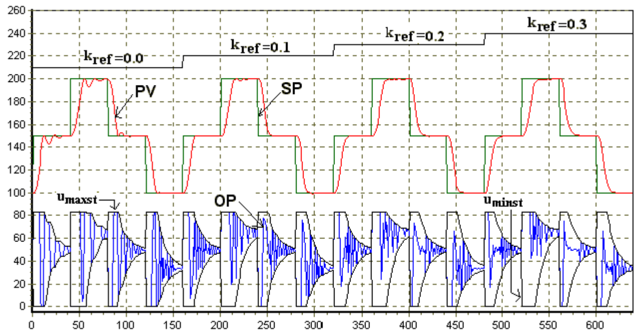

Fig. 8. Example 4

Example 5

Conditions: k_{st}=0.05, k_{ref}=0.1. In this example, a simple filter is used to reduce the variance of control signal in steady state regime, using next relation:

$$u(t) \leftarrow k_u \cdot u(t) + (1-k_u) \cdot u_{st} \quad (15)$$

where u_{st} is the estimated value of control signal in steady state regime and k_u is the filter parameter (Fig. 9).

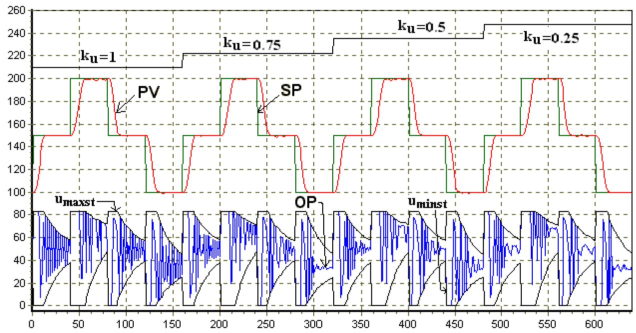

Fig. 9. Example 5

Example 6

In this example the static gain (k_0) of the process is variable from 1 to 1.6 with 0.2 step.

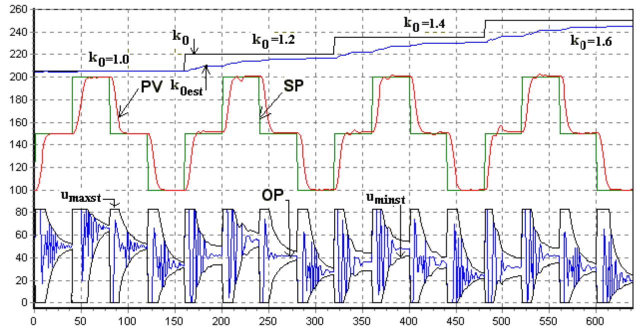

Fig. 10. Example 6

For identification, it is used a recursive least square algorithm. For control algorithm it is used model-based predictive control (MBPC-A1). The estimate of static gain is k_{0est}. The forgetting factor is λ=0.98, k_{st}=0.15, k_{ref}=0.2, noise: σ =0. If difference between process and model is quite larger, the control algorithm will compute a wrong control signal and it is possible to appear significant errors (for example at step 498 the override is 16% - Fig. 10). A method to reduce this effect is to choose cautions value for parameters, especially for k_{st} and k_{ref}.

IV. Application of the Algorithm in Thermal Building Control

Usually, based on building characteristics (dimensions of wall, windows, floor, ceiling, construction material parameters, etc.) are realized the thermal model of the building (according to the methodology presented in section II). This model can be used for realization of on-line simulations needed, thus based on the presented algorithm to be computed the control signal considered optimal. The thermal model presented takes into account the external temperature, the air exchange with exterior, the influence of other internal and external factors. In Figs. 11 and 12 are presented the results of the temperature control obtained based on the PID control (Fig. 12) as well on the model based control (Fig. 13). The value of setpoint for indoor temperature is 21º, the outdoor temperature is -20º, the sampling time is 1 minute, initial indoor temperature is 10º, the control signal is the proportion (0..1) of valve opening.

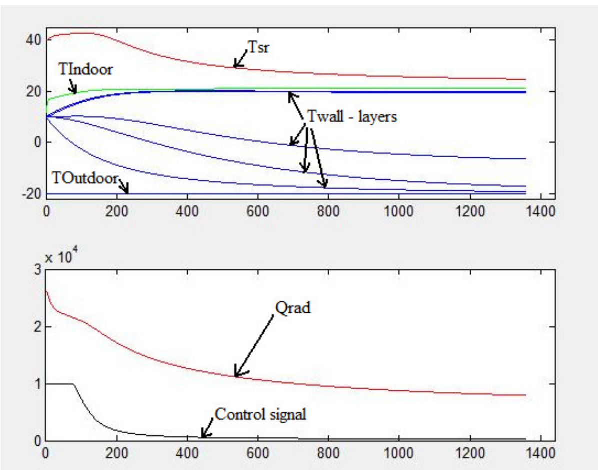

Fig. 12 PID Control

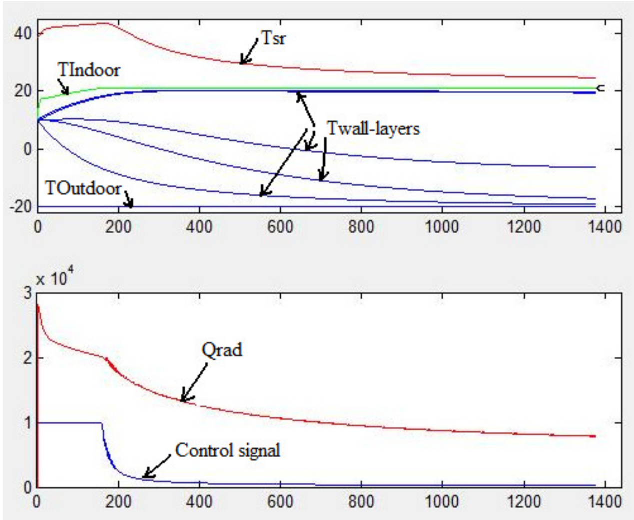

Fig. 13 MBPC Control

There aren't found major differences in the functioning of both algorithms, although the model based algorithm has a more rapidly response.

But in the case in which is aimed optimization of the energy consumption in conditions of realization of the desired comfort, the experience shows that model based control algorithm leads to better results [11].

Although, the methodology in which it is used the detailed thermal model of the building is difficult to be applied. On one hand, in the case of already built building, the needed data can be hard to be found. On the other hand there are situations in which the thermal characteristics are modified in time or due to the disturbing factors of which integration in the thermal model can be difficult or insufficient precisely (for example the solar radiation effect).

As a consequence, it was developed also solutions that take into account a lumped formulation of the model [12-14].

A solution which from the practical viewpoint would be simpler to be used (by avoiding the introduction of the model building parameters), as well as by implementation, is approximation of the building model with a linear parametric model and usage of on-line identification for renewal of the parameters.

Still, the effectuated simulations based on the usage of building model presented in the section 2 indicates the fact that, due to the nonlinearity process, this solution is difficult to be used and only for a constrained variation range of indoor temperature.

The main difficulty consists in the fact that sometimes exist the trend to obtain an inaccurate model and even instable, that will lead to weaker control performances.

In order to surpass these difficulties it was used parametric identification on ranges, respectively range 5-29° was divided in 8 subfields of 3 degrees (of course, from the practical viewpoint some subfields don't have relevance only for testing the method).

It was used a simple model, similarly with relation (14), with a_3=b_3=0. It was obtained the following results presented in table 1.

Table 1: Parameter identification

T°	a_1	a_2	b_1	b_2
5-8	-1.7182	0.7204	0.0226	-0.0049
8-11	-1.7361	0.7378	0.0212	-0.0055
11-14	-1.7638	0.7651	0.0188	-0.0062
14-17	-1.7770	0.7781	0.0177	-0.0064
17-20	-1.8056	0.8065	0.0155	-0.0069
20-23	-1.8111	0.8119	0.0151	-0.0070
23-26	-1.8539	0.8544	0.0126	-0.0079
26-29	-1.8823	0.8827	0.0117	-0.0086

More, for each range it can be build a bank of models, that will permit that at a certain moment, based on an performance index, to be chosen from the candidate models that model that is considered the best. The performance index can be chosen this way: for each model from the bank, it is simulated a number of steps previous to the process simulated behaviour and is compared with the real behaviour. A difficulty in construction of such a bank of models is represented by the choice of a criterion for introduction of some models in bank (a new model introduced in bank replaces the model considered the least fit and has to be sufficient "different" from the existent models in bank, otherwise there exist the trend that bank of models to contain models too similar, that reduces the flexibility of the method).

Using these methods, even if the computing time increases significantly, the control system results can improve significantly. In Figs. 14 and 15 are presented comparatively the results obtained in case of using the control algorithm PI respectively MBPC. In case of PI control algorithm, the tuning was realized for reference temperature of 21°C. It can be noticed a better behaviour in the case of control algorithm MBPC, both from the viewpoint of controller output as well as from the variation of the control signal.

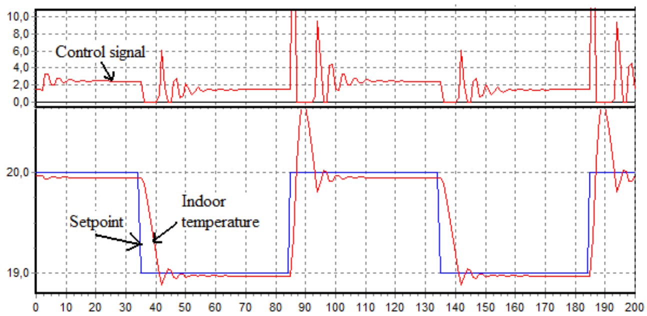

Fig. 14 PID Control

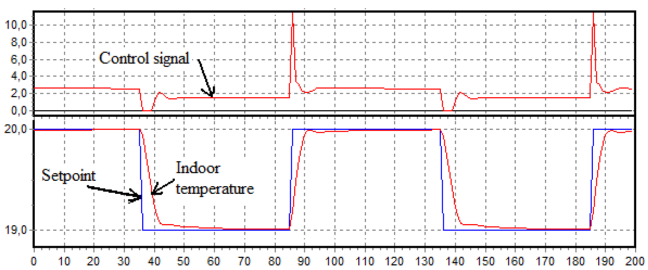

Fig. 15 MBPC Control

V. Conclusions

In the paper a model based control algorithm that can be applied to nonlinear systems was presented. The algorithm is applied for control of the temperature in a building and uses for predictions the discrete nonlinear model of the building. By the way it works the algorithm permits implementation of the optimization solutions of the energy consumption in conditions of maintaining a proper thermal comfort. The algorithm can be easy to realize in the case of the HVAC systems that is more difficult to control by means of classical PID control algorithms.

VI. Acknowledgment

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